An Optimal State Estimation Observer for Fault Detection of Gas Turbine Engine

Hamed Kazemi, and Alireza Yazdizadeh

Abstract— **This paper presents a new scheme based on state estimation to diagnosis an actuator or plant fault in a class of nonlinear systems that represent the nonlinear dynamic model of gas turbine engine. An optimal nonlinear observer is designed for the nonlinear system. By utilizing Lyapunov's direct method, the observer is proved to be optimal with respect to a performance function, including the magnitude of the observer gain and the convergence time. The observer gain is obtained by using approximation of Hamilton-Jacobi-Bellman (HJB) equation. The approximation is determined via an online trained neural network (NN). Using the proposed observer, the system states and the fault signal can be estimated and diagnosed, respectively. The proposed approach is implemented for state estimation and fault detection of a gas turbine model subject to compressor mass flow fault. The simulation results illustrate that the proposed fault detection scheme is a promising tool for the gas turbine diagnostics.**

***Index Terms*— Fault Diagnosis; Optimal State Estimation; Gas Turbine Engine; Nonlinear Observer Design, Neural Network.**

I. Introduction

OBSERVER design for nonlinear systems is a popular problem in control theory that has been investigated in many aspects. State estimation of nonlinear system is another interesting and relevant topic in the modern control theory; see [1], [2], and [3].In [1] considering multiobjective optimization an observer for state estimation of a class of uncertain nonlinear systems is designed. In [2] and [3] using linear methods state estimation of nonlinear system is investigated. As an important application area of observer design, model-based fault detection and isolation (FDI) is a well-established technique in literature [4]. So far, various observer-based FDI design approaches, including FDI via Kalman filter [5] and high gain adaptive observers [6], have been reported. Most of these techniques are developed for linear systems. However, during the past two decades, a number of observer-based FDI approaches for nonlinear systems have been presented; see [7], [8] and [9]. A review of nonlinear system model-based fault diagnosis has been done in [7]. An inversion-based fault reconstruction filter has been proposed in [8]. This filter is innovative and it is obtained by a geometric approach in which the filter design and related coordinate transformation are partially complicated. In [9] an observer-based fault estimation based on the nonsingular coordinate transformation is studied. This method is appropriate for a class of discrete Lipschitz nonlinear system.

The basic idea behind the use of the observer for fault detection is to estimate the states of the system by using some type of observers, and then construct a residual by a properly weighted output error, see [4] and [10]. For nonlinear systems, the theory of observer design is not nearly complete or successful, as it is for the linear case [11], and nonlinear observers related to FDI of nonlinear systems are restricted. Most of them use sliding mode approach for detecting or estimating the fault; see [12]. Designing and utilizing other observers in this field could be worthwhile.

Nonlinear observers are limited to a class of nonlinear systems and some of them use linearized model of the system [2]. It is also noted that most of the observer gains are very high or they depend on estimation error which is initially very high. However, high value of observer gain increases the sensitivity to noise. Also finite time convergence is an important feature that should be considered in designing procedure. In [11] according to a method presented in [13] for optimal control of nonlinear system, an optimal nonlinear observer using Hamilton-Jacobi-Bellman (HJB) equation based formulation is proposed for state estimation of a class of affine nonlinear system that is not subject to fault or unknown disturbance. Since it is difficult to find solution of the HJB equation, neural network (NN) has been used to approximate it. Here we want to use this method and consider it for a system subject to fault signal.

On the other hand, Fault diagnosis of aircraft gas turbine engines as a complex nonlinear system has received considerable interest in recent years due to the increasing demand on reliable operation and maintainability requirements of these safety critical systems [14]. Fault diagnosis methods are primarily divided into two main categories, namely model-base and data-driven techniques. Both of these techniques have been extensively studied in the literature for health monitoring of gas turbine engines [15], [16], [17] and [18]. The main framework of research in gas turbine engine FDI is based on

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gas path analysis (GPA) in which by measurement and estimation of lumped parameters of the system, such as temperature and pressure at each stage, one attempts to isolate and identify actuator, sensor, or component faults [19]. References to this approach first appeared in the literature by one of the early investigators and pioneers, Urban and Smetana [20]-[21]. Then it was developed mainly by Volponi and other researchers [22].

As a generality, diagnostic systems rely on discernable changes in observable parameters to detect physical faults. Physical faults consist of a variety of problems or combinations of problems such as foreign object damage, blade erosion and corrosion, worn seals, plugged nozzles, excessive blade tip clearances, etc. If severe enough, these physical faults will induce a change in the thermodynamic performance of the engine and its attendant components like efficiency and flow capacities of compressor and turbine [23] .The underlying precept behind GPA is that engine performance depends on the state of these individual components and that furthermore, the condition of these components can be mathematically represented by a set of independent performance parameters.

This paper contributes to the field of state estimation and fault detection of nonlinear systems. Firstly we propose an optimal nonlinear observer for nonlinear systems subject to a fault signal (actuator or plant fault) that in addition to estimating states, it is able to detect the occurrence of the fault. By utilizing Lyapunov's direct method, the observer is proved to be optimal with respect to a performance function including magnitude of observer gain and convergence time. Observer gain is obtained by using approximation of HJB equation that is determined via an online trained NN with time-varying weights. By utilizing the proposed observer, the system states and the fault signal can be estimated and diagnosed, respectively. Finally the proposed method is applied on gas turbine engine subject to compressor mass flow fault. In fact, the main innovation of the paper is designing the optimal nonlinear observer and implementing it for fault detection of gas turbine model.

In comparison to other methods related to the fault detection of nonlinear system, the approach proposed in this paper is effective for wide class of nonlinear system. For instance, methods based on sliding mode approach are limited to a class of nonlinear system in a standard form in which the nonlinear term, that is a separate term, must satisfies Lipschitz function assumptions. Also appearance chattering phenomenon in sliding mode observer is another restriction that should be avoided. These restrictions there are not in this work. Furthermore, here by utilizing a performance function, optimization of observer gain magnitude and finite-time state estimation are guaranteed.

The rest of this paper is organized as follows: In section 2, state estimation and fault detection scheme including observer design and NN based HJB solution, is presented. The implementation of the proposed approaches on gas turbine engine and the simulation results are provided in section 3. Finally, the conclusion remarks are given in section 4.

II. State Estimation and Fault Detection Scheme

A. *Nonlinear Observer Design*

Consider a nonlinear system of the form

$$\dot{x} = f(x,u) + g(x,u)\xi \qquad y = Cx \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^{n_u}$ is the control input, $y \in \mathbb{R}^d$ is the measured output, $\xi \in \mathbb{R}$ is the fault signal, $f(x,u)$ and $g(x,u)$ are smooth vector field and $C \in \mathbb{R}^{d\times n}$ is constant matrix. Consider a state observer with Luenberger like structure as follow

$$\dot{\hat{x}} = f(\hat{x},u) + L(y - C\hat{x}) \qquad e(t) = x(t) - \hat{x}(t) \tag{2}$$

where $\hat{x} \in \mathbb{R}^n$ is the estimated state vector, $e(t)$ is estimation error or residual, L is an observer gain matrix of appropriate dimension. Elements of L are nonlinear function of estimation error and they are bounded by a positive constant λ, i.e. $L_{ij} \leq \lambda \in \mathbb{R}$. The estimation error dynamics may be computed as

$$\dot{e} = f(x,u) - f(\hat{x},u) + g(x,u)\xi - LCe. \tag{3}$$

The problem of state estimation and, consequently, residual generation is to design an observer, modeled by equations of the form (1), such that the residual e with dynamic equation (3) asymptotically converges to zero in absence of fault ξ. So assuming $g(x,u) \neq 0$ the requirements of Fundamental Problem of Residual Generation (FPRG), i.e. the residual eonly is affected by fault ξ, is satisfied.

The main challenge of designing observer (1) is to determine gain matrix L such that the convergence criterion of residual e is satisfied. The amount of mentioned gain and finite-time convergence are important attributes that should be considered. In [13], a procedure for optimal designing of a controller is presented. By utilizing that the method of optimal observer design for fault detection, is addressed as follows:

For the system (3) find an observer gain L such that the following finite-horizon performance function with its terminal cost is minimized.

$$\Phi(e,t) = \int_{t_0}^{t_f} \left(e^T Q e + K(L)\right) dt \tag{4}$$

where

$$K(L) = 2\int_0^L \tanh^{-1}\left(\frac{v}{\lambda}\right) dv. \tag{5}$$

Where λ is a coefficient, v is an auxiliary variable and $\int_0^L \tanh^{-1}\left(\frac{v}{\lambda}\right) dv \equiv \int_0^{L_{11}} \tanh^{-1}\left(\frac{v_{11}}{\lambda}\right) dv_{11} + \int_0^{L_{12}} \tanh^{-1}\left(\frac{v_{12}}{\lambda}\right) dv_{12} + \cdots + \int_0^{L_{dn}} \tanh^{-1}\left(\frac{v_{nd}}{\lambda}\right) dv_{nd}$ is a non-quadratic term expressing cost related to constrained observer gain in which $\int_0^{L_{ij}} \tanh^{-1}\left(\frac{v_{ij}}{\lambda}\right) dv_{ij} = 2L_{ij}\lambda \tanh^{-1}\left(\frac{L_{ij}}{\lambda}\right) + \lambda^2 \ln(1 - {L_{ij}}^2/\lambda^2) > 0$ and L_{ij} is the element of matrix L. The existence of estimation error e with positive definite matrix Q, observer gain L and final convergence time t_f in performance function (4), lead to the convergence properties of estimation error present in optimal observer design.

To solve the constrained optimal nonlinear observer design, let

$$V(e_0,t_0)=\Phi\big(e(t_f),t_f\big)+\min_L\int_{t_0}^{t_f}(e^TQe+K(L))dt \tag{6}$$

be the minimum cost of bringing the system (3) from initial condition e_0 to equilibrium point 0.

Definition 1 (Admissible observer gain): An observer gain L is defined to be admissible with respect to (6) on Ω, denoted by $L\in\psi(\Omega)$ with L continuous on Ω, if it stabilizes (3) for $\xi=0$ on Ω, and for $\forall e_0\in\Omega$,, $V(e_0,t_0)$ is finite.

Under regularity assumptions, i.e. $V(e,t)\in C^1(\Omega)$, considering (6) the HJB gives

$$\min_L\big(e^TQe+K(L)+V_t+V_e^T(f(x,u)-f(\hat{x},u)+g(x,u)\xi-LCe)\big)=0 \tag{7}$$

where $V_t=\frac{\partial V}{\partial t}$ and $V_e=\frac{\partial V}{\partial e}$. This is a time-varying partial differential equation (PDE) with $V(e,t)$ that is the cost function for any given L and it is solved backward in time from $t=t_f$. By setting $t_0=t_f$ for (6) we have $V(e(t_f),t_f)=\Phi(e(t_f),t_f)$.

If L is the solution to the optimal observer design problem, then according to Bellman's optimally principle [24], the optimal cost is given by

$$\mathrm{HJB}\big(V(e,t)\big)=e^TQe+K(L)+V_t+{V_e}^T(f(x,u)-f(\hat{x},u)+g(x,u)\xi-Lce)=0. \tag{8}$$

Optimal observer gain matrix, L, can be derived by solving $\partial\Big(\mathrm{HJB}\big(V(e,t)\big)\Big)/\,\partial L=0$. Using (8) and (5), this equation can be written as:

$$L=\lambda\tanh\Big(\frac{1}{2\lambda}V_e(Ce)^T\Big) \tag{9}$$

where $V(e,t)$ is the optimum value function. The time-varying observer gain matrix (9) represents constrained dynamic optimal observer for the nonlinear systems. The validity of an optimal observer design is expressed in the next theorem.

Theorem .1: Consider the nonlinear system (3) and performance function (6). Assume that there exists a function $V(e,t)$ as the solution of HJB equation (8). The observer (1) acts as an optimal residual generator that estimate x if no fault has occurred ($\xi(t)=0$), addressing constraints with respect to terminal time and the observer gain. If a fault has occurred ($\xi(t)\neq 0$), the estimate of x is such that $\|e\|>\epsilon$, where ϵ is a positive constant.

Proof: Let us first study the case without fault; we show that L is a solution to the optimal observer design problem, i.e. the residual e in system (3) converges to zero globally asymptotically, which can be proved by showing $V(e,t)$, the solution of HJB equation (8), is a Lyapunov function. Clearly, $V(e,t)>0$ for $\forall e\neq 0$ and $t\neq 0$\$and $V(0)=0$ also considering (8) we have

$$\dot{V}(e,t)=\frac{\partial V}{\partial t}+\Big(\frac{\partial V}{\partial e}\Big)^T\dot{e}=-e^TQe-K(L)\le -e^TQe\le 0 \tag{10}$$

consequently, there exists a neighborhood $Z=\{e:\|e\|<\epsilon\}$ for some $\epsilon>0$ such that if e(t) enters Z, then $\lim_{t\to\infty}e(t)=0$. But e(t) cannot remain forever outside Z. Otherwise $\|e\|\ge\epsilon$, for all $t\ge 0$. Let $\alpha=\inf(e^TQe)>0$ such that $\|e\|\ge\epsilon$.Therefore, $V(e(t),t)-V(e(0),0)=\int_0^t\dot{V}(e(\tau),\tau)d\tau\le-\int_0^t\alpha d\tau=-\alpha\int_0^t d\tau=-\alpha t$. Let $t\to\infty$, we have,

$$V(e(t),t)\le V(e(0),0)-\alpha t\to-\infty$$

which contradicts the fact that $V(e,t)>0$ for $\forall e\neq 0$.

Therefore $\lim_{t\to\infty}e(t)=0$, no matter where the trajectory begins. It concludes the optimal stability of the state estimation when no fault has occurred.

Let us now discuss the case when a fault has occurred. From (3) one can see that for $g(x,u)\neq 0$ the time derivative of the estimation error is directly influenced by the fault. Since a fault is detectable, its occurrence causes a change in nominal behavior of system. So $\|e\|>\epsilon$.

Hence one can design a constrained optimal observer using proposed formulation for nonlinear systems with finite time horizon. An optimal observer can be designed by knowing exact solution of HJB equation, which is a difficult problem. In [11] utilizing a NN which is proposed in [13], the solution of HJB equation is approximated for calculating an observer gain related to an affine nonlinear system. In the next section, according to that NN, the approximation of value-function V which is the solution of HJB equation is obtained.

B. NN Based HJB Solution

In this section, for finding approximate solution of HJB equation we use NN. In [25], it is shown that an online trained NN with time-varying weights can be used to approximate smooth time-varying functions on prescribed compact sets. Actually the approximate solution is used to find observer gain. Therefore assuming that $V(e,t)$ is smooth and also uniformly continuous on a compact set Ω, one can use the following equation to approximate $V(e,t)$ for $t\in[t_0,t_f]$ on Ω.

$$\hat{V}(e,t)=\sum_{J=1}^{N}w_j(t)\sigma_j(e)=W^T(t)\Xi(e) \tag{11}$$

This is a NN with activation function $\sigma_j(e)\in C^1(\mho)$ and $\sigma_j(0)=0$. We have

$$\hat{V}_e(e,t)=\frac{\partial\hat{V}}{\partial e}=\big(\nabla\Xi(e)\big)^TW(t) \tag{12}$$

and

$$\hat{V}_t(e,t) = \frac{\partial \hat{V}}{\partial t} = \Xi^T(e)\frac{\partial W(t)}{\partial t} \tag{13}$$

where $w_j(t)$ denotes the NN weight and N is the number of hidden layer neurons, $\Xi(e) = [\sigma_1(e)\ \sigma_2(e)\ \dots\ \sigma_N(e)]^T$ is the vector of activation function selected such that $\hat{V}(0) = 0$ and $\hat{V}(e,t) > 0$ for $\forall e \neq 0$ and $t \neq 0$ and $W = [w_1\ w_2 \dots w_N]^T$ is the vector of NN weights. It is assumed that N is large enough that there exist weight $W(t_f)$ that exactly satisfy the approximation at $t = t_f$. Without loss of generality, the set $\Xi(e)$ is selected to be independent and orthonormal [13]. The orthonormality of the set $\{\sigma(e)\}_1^\infty$ on $\Omega \subseteq R^n$ imply that, for a real-valued function $\Theta(e,t) \in R$,,

$$\Theta(e,t) = \sum_{J=1}^{\infty} \langle \Theta(e,t), \sigma_j(e)\rangle_{\mho}\sigma_j(e)$$

where $\langle f,g\rangle_{\mho} = \int_{\mho} a.b^T dx$ is an inner product, a and b are continuous vector functions, and the series converge pointwise [26], i.e. for any $\varepsilon > 0$ and $e \in \Omega$, one can choose N sufficiently large to guarantee that $\left|\sum_{J=N+1}^{\infty}\langle \Theta(e,t), \sigma_j(e)\rangle_{\mho}\sigma_j(e)\right| < \varepsilon$ for all time $t \in [t_0, t_f]$.

Note that $\frac{\partial V}{\partial t}$ in (8) is required, so the NN weights are selected to be time varying. Approximating $V(e,t)$ by $\hat{V}(e,t)$ in the HJB equation (8) results in

$$HJB\left(\hat{V}(e,t)\right) = e^T Q e + K(\hat{L}) + \hat{V}_t + V_e^T\big(f(x,u) - f(\hat{x},u) + g(\hat{x},u)\xi - \hat{L}Ce\big) = E \tag{14}$$

where E is the approximation error. If E is negligible, then (14) is similar to (8).

Assuming the fault $\xi(t)$ and coefficients $|w_j(t)|$ for all N are uniformly bounded, the following lemma shows the existence of NN based HJB solution for optimal observer design using performance function (6).

Lemma 1: Given $L \in \psi\ (\mho)$, let $\hat{V}(e,t) = \sum_{j=1}^{N} w_j(t)\sigma_j(e)$ satisfy $\langle HJB\left(\hat{V}(e,t)\right), \Xi(e)\rangle_{\mho} = 0$ and $\langle \hat{V}(t_f), \Xi(e)\rangle_{\mho} = 0$, and let $V(e,t) = \sum_{j=1}^{\infty} b_j(t)\sigma_j(e)$ and $B = [b_1(t)\ \ b_2(t) \dots b_N(t)]^T$ satisfy $HJB\big(V(e,t)\big) = 0$ and $V(e(t_f), t_f) = \phi(e(t_f), t_f)$, then $\left|HJB\left(\hat{V}(e,t)\right)\right| \to 0$ uniformly on Ω as N increases.

There is a theorem for the existence of NN based HJB solution for the optimal control problem in [13]. The existence of NN based HJB solution for optimal observer using modified performance functional can be proved on similar lines. See [13].

Since Lemma 1 shows the existence of NN based HJB solution, (14) can be written as

$$HJB\left(\hat{V}(e,t)\right) = e^T Q e + K(\hat{L}) + \hat{V}_t + V_e^T\big(f(x,u) - f(\hat{x},u) + g(\hat{x},u)\xi - \hat{L}Ce\big) \approx 0\ . \tag{15}$$

In the next theorem we prove that the nonlinear observer (1) is an optimum observer that estimates states in absence the fault and detects the fault when it occurs. It also proves the validity of the NN-HJB based observer design.

Theorem 2: Consider the error dynamics (3) with the performance function (6). Assume that there exists a function $\hat{V}(e,t)$ as the solution of HJB equation (14). Using this solution, if no fault has occurred observer gain matrix $L = \hat{L}$ ensures global asymptotic stability of system (3), i.e., error $e = x - \hat{x}$ asymptotically converges to zero. If a fault has occurred, $\|e\| > \epsilon$, where ϵ is a positive constant.

Proof: Using (12) and (15), we can find approximate optimal observer gain matrix similarly as in (8) by following equation:

$$\hat{L} = \lambda \tanh\left(\frac{1}{2\lambda} V_e (Ce)^T\right) = \lambda \tanh\left(\frac{1}{2\lambda}\nabla\Xi^T(e)W(t)(Ce)^T\right). \tag{16}$$

Vector $\Xi(e)$ can be selected such that $\hat{V}(0) = 0$ and $\hat{V}(e,t) > 0$ for $e \neq 0$. Also $\dot{\hat{V}}(e,t) < 0$ for $e \neq 0$ can be proved similarly as Theorem 1 by replacing $V(e,t)$ with $\hat{V}(e,t)$. So replacing L with $\hat{L}$, the system (3) remains globally asymptotically stable. Hence it can be proved that $\hat{V}(e,t)$, the solution of HJB equation (15) is a Lyapunov function.

From the above theorem we can say that an optimal observer with gain matrix (16) can be designed for a nonlinear system using HJB formulation. For calculating gain matrix, set of the NN weights is required. We describe this in the following

The NN weights are selected to minimize approximation error in least square sense over a set of points sampled from a compact set Ω_0 inside the region of stability of the initial stabilizing control [13]. To find the least squares solution, the method of weighted residuals is used. This method was explored in [27] for optimal control problem based on HJB formulation. On similar lines, one can explore this method for observer design problem. The weights W are determined by projecting the residual error onto $dE/d\dot{W}$ and setting the result to zero $\forall e \in \Omega$ using the inner product, i.e.

$$\langle \frac{dE}{d\dot{W}}, E\rangle = 0 \tag{17}$$

where $\langle a,b\rangle = \int_{\Omega} ab dx$ is a Lebesgue integral. According to this method, by using equations (11) - (14), we have

$$\frac{\partial E(e,t)}{\partial \dot{W}(t)} = \Xi(e) \tag{18}$$

Then equation (18) can be written as

$$\langle e^T Q e, \Xi(e)\rangle_{\Omega} + \langle K(\hat{L}), \Xi(e)\rangle_{\Omega} + \langle \Xi^T(e)\dot{W}(t), \Xi(e)\rangle_{\Omega}$$
$$+\langle W^T(t)\nabla\Xi(e)(f(x,u) - f(\hat{x},u) + g(\hat{x},u)\xi), \Xi(e)\rangle_{\Omega}$$
$$-\langle W^T(t)\nabla\Xi(e)\lambda\tanh\left(\frac{1}{2\lambda}\nabla\Xi^T(e)We^TC^T\right)Ce, \Xi(e)\rangle_{\Omega} = 0 \tag{19}$$

Hence weight updating law can be obtained as

$$\dot{W}(t) = -\langle \Xi(e), \Xi(e) \rangle_{\Omega}^{-1} \{ \langle \nabla\Xi(e)(f(x,u) - f(\hat{x},u) + g(\hat{x},u)\xi), \Xi(e) \rangle_{\Omega} W(t) - \langle e^T Qe, \Xi(e) \rangle_{\Omega} - \langle K(\hat{L}), \Xi(e) \rangle_{\Omega} - \langle W^T(t) \nabla\Xi(e) \lambda \tanh\left(\frac{1}{2\lambda} \nabla\Xi^T(e) W e^T C^T\right) Ce, \Xi(e) \rangle_{\Omega} \} \quad (20)$$

The NN weights can be determined by integrating (20) backwards in time using final condition $W(t_f)$. Observer gain (16) can be found using these weights.

III. Simulation Results

A. Gas Turbine Dynamic Model

Based on the available literature on modeling of gas turbine engines [28], [29], a SIMULINK model for a commercial single spool gas turbine engine at normal operating conditions is considered. A more detailed description of the model can be found in Refs. [28]-[30]. The set of nonlinear equations corresponding to a single spool gas turbine engine is given by

$$\dot{T}_{CC} = a_{11} T_C \dot{m}_C + a_{12} \dot{m}_f - a_{13} T_{CC} \dot{m}_T - a_{14} T_{CC} (\dot{m}_C + \dot{m}_f - \dot{m}_T) + (a_{11} T_C - a_{14} T_{CC}) \xi$$

$$\dot{N} = \frac{a_{21} \dot{m}_T (T_{CC} - T_T) + a_{22} \dot{m}_C (T_d - T_C) + a_{22} (T_d - T_C) \xi}{N}$$

$$\dot{P}_T = a_{31} T_M (\dot{m}_T - \dot{m}_n) + a_{32} T_M \dot{m}_C + a_{32} T_M \xi$$

$$\dot{P}_{CC} = a_{41} \frac{P_{CC}}{T_{CC}} T_C \dot{m}_C + a_{42} \frac{P_{CC}}{T_{CC}} \dot{m}_f - a_{43} P_{CC} \dot{m}_T - (a_{44} P_{CC} - a_{45} T_{CC})(\dot{m}_C + \dot{m}_f - \dot{m}_T) + \left(a_{41} \frac{P_{CC}}{T_{CC}} T_C - (a_{44} P_{CC} - a_{45} T_{CC})\right) \xi$$

$$\frac{d\dot{m}_f}{dt} = -a_{51} \dot{m}_f + a_{52} u_{fd} \quad (21)$$

where $a_{11} = a_{13} = a_{41} = a_{43} = \frac{c_p}{c_v m_{CC}}$, $a_{12} = \frac{\eta_{CC} H_u}{c_v m_{CC}}$, $a_{14} = \frac{1}{m_{CC}}$, $a_{21} = \frac{\eta_{mech} c_p}{J(\pi/30)^2}$, $a_{22} = \frac{c_p}{J(\pi/30)^2}$, $a_{31} = \frac{R}{V_M}$, $a_{32} = \frac{R}{V_M} \frac{\beta}{1+\beta}$, $a_{42} = \frac{\eta_{CC} H_u}{c_v m_{CC}}$, $a_{44} = c_v$, $a_{45} = \frac{\gamma R}{V_{CC}}$, $a_{51} = \frac{1}{\tau}$ and $a_{52} = \frac{G}{\tau}$; in which T_{CC}, T_C, T_T and T_M denote the combustion chamber, compressor, turbine and mixer temperatures, respectively, P_{CC}, P_C , and P_T denote the combustion chamber, compressor and turbine pressures, respectively, V_M, V_{CC}, denote the volume of gas inside mixer and combustion chamber, respectively, N denotes the rotational speed, $\dot{m}$ is the mass flow in different components, η_{mech} denotes the mechanical efficiencies, J denotes the inertia of the shaft, m_{CC} denotes the mass of air inside the combustion chamber, η_{CC} denotes the combustion chamber efficiency, β denotes the bypass ratio, γ denotes the heat capacity ratio, R denotes the gas constant, c_v denotes the heat at constant volume, c_p denotes the specific heat at constant pressure, and H_u denotes the fuel specific heat. τ is the time constant, G is the gain, and u is the fuel demand that is computed by using a feedback from the rotational speed as described in [28].

The state variables and the output measurement in the gas turbine engine are selected as $x = [T_{CC}, N, P_T, P_{CC}, \dot{m}_f]^T$ and $y = [N, P_T]^T$. A modular SIMULINK model is developed to simulate the above-mentioned gas turbine engine nonlinear dynamics. Fig. 1 shows the information flow process in our SIMULINK model of the engine.

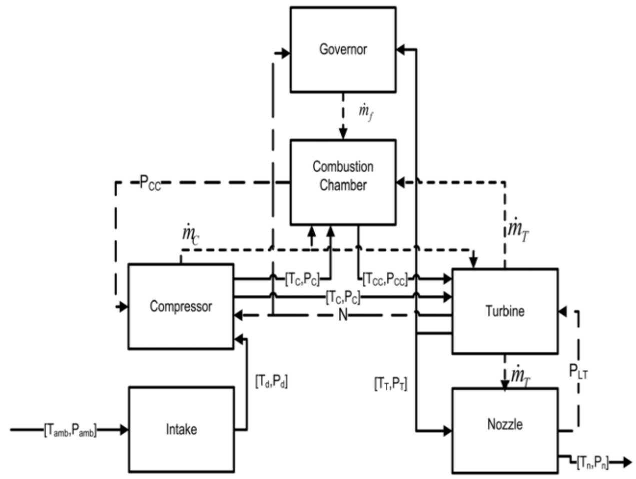

Fig. 1 Information flow diagram in a modular modeling of the gas turbine engine dynamics [31]

32In this brief, a component anomaly is considered as sources of fault. Component fault is modeled as changes in the compressor mass flow with respect to the normal mode. Referring to [28] one can find more details about this fault. This fault is denoted by ξ. Paying attention to the presented method of fault detection, the aim of this section is to detect the occurrence of that utilizing the optimal state estimation.

B. Optimal State Observer

Now utilizing the presented observer we want to estimate measured system states and diagnosis the fault. Regard system (21) using (16), we can calculate L and an observer in the form (1), residual error for this system is

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} \hat{x}_2 \\ \hat{x}_3 \end{bmatrix}$$

We have to find a nonlinear observer gain L that minimizes (6). Here we have selected

$$V(e,t) = w_1(t)e_1^2 + w_2(t)e_2^2 + w_3(t)e_1e_2 + w_4(t)e_1^4 + w_5(t)e_2^4 + w_6(t)e_1^3e_2 + w_7(t)e_1^2e_2^2 + w_8(t)e_1e_2^3 + w_9(t)e_1^6 + w_{10}(t)e_2^6 + w_{11}(t)e_1^5e_2 + w_{12}(t)e_1^4e_2^2 + w_{13}(t)e_1^3e_2^3 + w_{14}(t)e_1^2e_2^4 + w_{15}(t)e_1e_2^5 \quad (22)$$

This is a NN with polynomial activation function. It is a power series NN of 15 activation functions containing powers up to 6th order of the error variable of the system. An appreciable change in the result is not observed if we use power series with order of 7 or more. It is also observed that for the power series up to 5th order, algorithm did not converge. So, design is carried out using the above mentioned. In the next part for simulation, we have $t_f = 50$ s and $W(t_f)$ is as follows

$$W(t_f) = [1\ \ 6\ \ 2\ \ 5\ \ 1\ \ 1\ \ 6\ \ 2\ \ 6\ \ 1\ \ 1\ \ 5\ \ 2\ \ 5\ 1]$$

Also $|L| \leq 6$ and $Q = [1000\ \ 0; 0\ 1000]$..

C. Simulation

For the system, it is assumed that the ambient conditions are set to standard conditions. The parameters corresponding to the model (21) are selected as follows: $J = 8\ \mathrm{Kg\ m^2}$, $V_M = 0.45\ \mathrm{m^3}$, $V_{CC} = 0.2\ \mathrm{m^3}$ $\eta_{CC} = 0.994$ $\eta_{mech} = 0.995$ $H_u = 48830\ \mathrm{J\ kg}$ $\beta = 0.62$, $\gamma = 1.4$ $R = 287 \mathrm{J/KgK}$ $c_v = 717\ \mathrm{J/Kg\ K}$ and $c_v = 1004\ \mathrm{J/KgK}$. We consider a simulation corresponding to the following scenario: the actuator is supposed to provide constant value equal to $u = 0.4$. At time $t = 15$ s an incipient fault (the component fault ξ) occurs. Fig. 2a - b, respectively present the observed variables N, P_T of the fault-free and faulty system. Also Fig. 2c shows the applied fault. The simulation results of the optimal state estimation and fault detection are shown in the next two figures.

As mentioned in section 2, the gain of the observer is obtained by minimizing the performance function (4) contained the estimation error, a function of observer gain and the convergence time. In fact without this performance function, convergence of the estimation error and thus proper operation of estimator in absence of the fault signal is not guaranteed. Furthermore, this function causes the observer gain to not exceed a fixed value. The simulation results confirm these expectations. Fig.3 demonstrates the actual and estimated responses of the states N, P_T in which estimation is result of an observer in the form (1) related to the system (21). Also Fig. 4 presents residual error e of system (21) and observer gain L corresponding to fault ξ. It can be highlighted that the estimation error e converges towards zero when the fault ξ doesn't occur. When it appears at $t = 15$ s, the residual signal shows an abnormal behavior of the system therefore the occurrence of ξ is detectable.

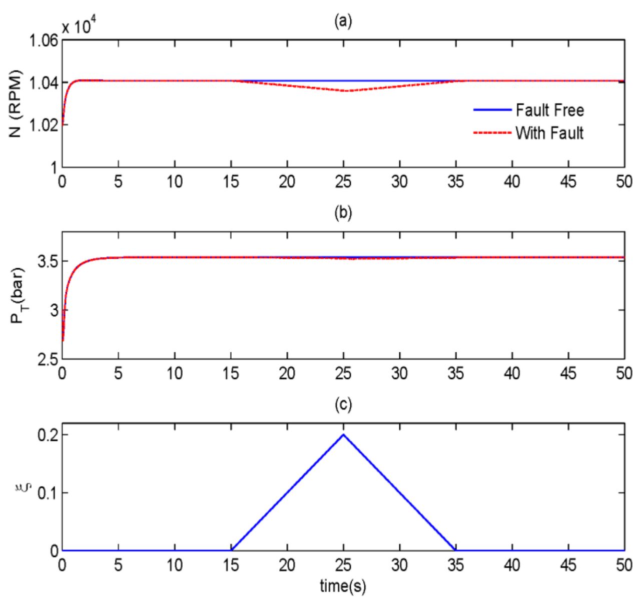

Fig. 2 Output of gas turbine model (fault free and faulty mode); and [(c)]: applied fault ξ

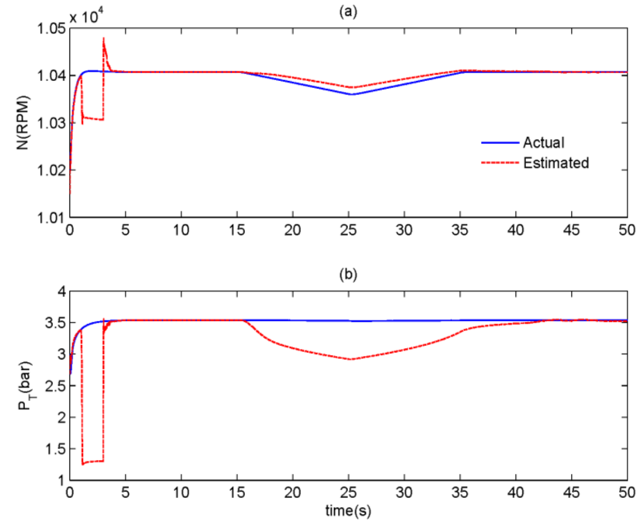

Fig. 3 Estimated states of gas turbine model

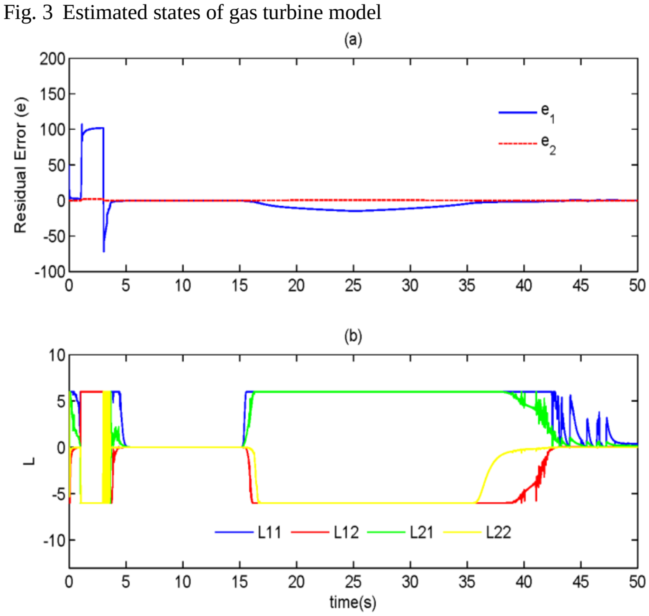

Fig. 4 Residual error and observer gain

Considering Fig. 4b, variation of each element of observer gain L during fault occurence is sensible. It is clear from these figures that the observer is able to track the true state of the system if no fault is occurred.

IV. Conclusion

In this paper we presented an approach for state estimation and fault detection in nonlinear systems in basis of optimal observer design. We brought out an optimal state estimator for nonlinear systems subject to an actuator or plant fault. By utilizing Lyapunov's direct method, the observer is proved to be optimal with respect to a performance function including magnitude of observer gain and convergence time. The approach proposed in this paper is effective for wide class of nonlinear system and the nonlinear term does not need to be a separate term, also appearance chattering phenomena is avoided. Finally, design procedure and performance of the avoided. Finally, design procedure and performance of the proposed scheme were illustrated through implementation of an observer for state estimation and fault detection of compressor mass flow fault of gas turbine model. The simulation results.

REFERENCES

[1] S. Delshad, A. Johansson, M. Darouach, and T. Gustafsson, “Robust state estimation and unknown inputs reconstruction for a class of nonlinear systems: Multi objective approach,” Automatica, vol. 64, pp. 1-7, 2016.

[2] W. Chen, and M. Saif, “Unknown input observer design for a class of nonlinear systems: an LMI approach,” American control conference, pp. 834-838, Vancouver, 2006.

[3] E. A. Boroujeni, H. R. Momeni and M. Sojoodi, “An LMI Approach to Resilient H Fractional Order Observer Design for Lipschitz Fractional Order Nonlinear Systems Using Continuous Frequency Distribution”, Modares Journal Of Electrical Engineering ,vol.12, no.1, pp. 1-10, 2015.

[4] J. Chen, and R. Patton, “Robust Model-Based Fault Diagnosis for Dynamic Systems”, Springer Berlin, 2009.

[5] L. Li, Z. Wang and Y. Shen, “Fault diagnosis for attitude sensors via a bank of extended Kalman filters”, 35th Chinese Control Conference , pp. 824 – 831, 2016.

[6] A. Xu, and Q. Zhang, “Residual generation for fault diagnosis in linear time-varying systems”, IEEE Trans. Autom. Control, vol. 49, no. 5, pp. 767 – 772, 2004.

[7] R. Isermann, “Model-based fault detection and diagnosis-status and applications”, Annu. Rev. Control, vol. 29, pp.71 – 75, 2005.

[8] H. Kazemi, and A. Yazdizadeh, “Fault reconstruction in a class of nonlinear systems using inversion-based filter”, Nonlinear dynamics, vol. 85, no. 3, pp. 1805-1814, 2016.[1]

[9] H. Qian, Y. Peng, and Y. Guangyi “Reduced-order observer-based fault estimation and fault-tolerant control for a class of discrete Lipschitz nonlinear systems”, Optimal Control Applications and Methods, 2016; nder press.

[10] M. Du, and P. Mhaskar, “Isolation and handling of sensor faults in nonlinear systems”, Automatica, vol. 50, no. 4, pp. 1066 – 1074, 2014.

[11] D. M., Adhyaru, “State observer design for nonlinear systems using neural network”, Appl. Soft Comp., vol. 12, pp. 2530 – 2537, 2012.

[12] J.V Gorpl, M. Defoort, M., Djemai, “Fault detection based on higher-order sliding mode observer for a class of switched linear systems”, IET Control Theory Appl., vol. 9, no. 15, pp. 2249 – 2256, 2015.

[13] T. Cheng, F. L. Lewis, and M. Abu-Khalaf, “Fixed final time constrained optimal control of nonlinear systems using neural network HJB Approach”, IEEE Trans. Neural Network, vol. 18, no. 6, pp. 1725 – 1737, 2007.

[14] S. Sina, B. Tayarani, K. Khorasani, “Fault detection and isolation of gas turbine engines using a bank of neural networks”, J. Process Control, vol. 36, pp. 22—41, 2015.

[15] N. Meskin, E. Naderi, K. Khorasani,“A multiple model-based approach for fault diagnosis of jet engines”, IEEE Trans. Control Syst. Technol., vol. 21, pp. 254—262, 2013.

[16] H. Asgari, X.Q. Chen, M.B. Menhaj, and R. Sainudiin, “Ann-based system identification, modelling and control of gas turbines – a review”, Adv. Mater. Res.,vol. 6, pp. 611–-617, 2012.

[17] R. Jiang,“A pnn fault diagnosis method for gas turbine”, in: 2012 World Automation Congress (WAC), pp. 1-4, 2012.

[18] Y. P. Cao, S. Y. Li, S. Yi and N. Zhao, “Fault diagnosis of a gas turbine gas fuel system using a self-organizing network”, Adv. Sci. Lett., vol. 7, 386-392, 2012.

[19] J. S. Litt, D. L. Simon, and S. Garg,“A Survey of Intelligent Control and Health Management Technologies for Aircraft Propulsion Systems”, J. Aeros. Comput. Inf. Comun., vol. 12, pp. 543–-563, 2004.

[20] L. A. Urban, “Modelling Methods for Improving Robustness in Fault Diagnosis of Jet Engine System”, in: 1972 Proceedings of the 8th ASME Joint Propulsion Specialist Conference, pp. 1-8, New York, 1972.

[21] F. Smetana, “Turbojet Engine Gas Path Analysis—A Review”, AGARD-CP-165, 1974.

[22] A. J. Volponi,“Foundations of Gas Path Analysis I \& II (von Karman Lecture Notes), von Karman Institute, Rhode-Saint-Genese, Belgium, 2003.

[23] A. J. Volponi, “Gas Turbine Engine Health Management: Past, Present, and Future Trends”, Journal of Engineering for Gas Turbines and Power, vol. 136, pp. 051201-20, 2014.

[24] M. Gopal,“Modern Control System Theory”, New Age International Publishers, 1993.

[25] WE, Sandberg, “Notes on uniform approximation of time-varying systems on finite time intervals”, IEEE Trans. Circuits Syst. I. Funda. Theory, vol. 45 no. 8, pp. 863 – 865, 1998

[26] R. Beard, G. Saridis, and J. Wen,“Galerkin approximations of the generalized Hamilton–Jacobi–Bellman equation”, Automatica, vol. 33 pp. 2159 – 2177, 1997.

[27] M. Abu-Khalaf, J. Huang, FL. Lewis, “Nonlinear H2/H Constrained Feedback Control: A Practical Design Approach using Neural Networks”, Springer, 2006.

[28] S. M. Camporeale, B. Fortunato, and M. Mastrovito,“A Modular Code for Real Time Dynamic Simulation of Gas Turbines in SIMULINK”, J. Eng. Gas Turbines Power, vol. 128, pp. 506—517, 2006.

[29] V. Panov, and Gasturbolib,“Simulink library for gas turbine engine modeling”, In: ASME Conference Proceedings Turbo Expo 2009, pp.555—565, Orlando 2009.

[30] R. Mohammadi, E. Naderi, K. Khorasani, and S. Hashtrudi-Zad, “Fault Diagnosis of Gas Turbine Engines by Using Dynamic Neural Networks” ,Turbo Expo 2010, ASME Conference Proceedings, Scotland, pp. 365—376, 2010.

[31] E. Naderi, N. Meskin, K. Khorasani, “Nonlinear Fault Diagnosis of Jet Engines by Using a Multiple Model-Based Approach”, Journal of Engineering for Gas Turbine, vol. 134, pp. 134-141, 2011.