Mathematical Modeling of Forced Convection Thin Layer Solar Drying for *Cuminum cyminum*

A. Zomorodian* and M. Moradi

ABSTRACT

This paper presents mathematical models of thin layer forced convection solar drying of *Cuminum cyminum* using two drying methods (mixed and indirect) at different operating conditions. The average initial moisture content of the seeds for all tests was about 43% d.b. and the drying was performed continuously, in each test, for a uniform period of 90 minutes drying time in a solar cabinet dryer to obtain an average final moisture content of 8% d.b. Three airflow rates (0.084, 0.127 and 0.155 m$^3$s$^{-1}$) were adopted and the experiments were run each sunny day from 11:30 to 13:00 with an average solar intensity of 750 W m$^{-2}$ (±50 W m$^{-2}$), ambient air temperature of 27°C (±1°C) and relative humidity of 30% (±1%). In order to find the most suitable form of thin layer solar drying model, eleven different mathematical models were selected using the experimental data to determine the pertinent coefficients for each model by applying the non-linear regression analysis technique. The goodness of fit was evaluated by calculating and comparing the statistical values of the coefficient of determination ($R^2$), reduced chi-square ($\chi^2$) and root of mean square error (RSME) for any model and for the two drying methods. The best results were found for the approximation of diffusion model with $R^2 = 0.995$, $\chi^2 = 0.0023$ and RSME = 0.0199 in mixed mode type, and the Midilli model with $R^2 = 0.994$, $\chi^2 = 0.0045$ and RSME = 0.0225 in indirect mode type thin layer solar drying.

Keywords: *Cuminum cyminum*, Mathematical model, Thin layer drying.

INTRODUCTION

Drying of agricultural products may be one of the most important unit operations for the preservation of food materials [14, 16]. Diminishing reserves of fossil fuels and increased costs have led to a search for alternative energy sources including solar energy for drying agricultural products [5, 12, 16, 17, 23]. Open-sun drying used to be an appropriate means in many urban and rural areas, but this conventional method cannot protect food materials from rain, dust, the attack by insects, birds and other animals. Therefore, it may increase the loss of products and have some adverse economic impacts on them [12]. Solar drying is a well-known food preservation procedure used to reduce the moisture content of agricultural products, which reduces quality degradation over an extended storage period [10].

*Cuminum cyminum* (cumin), a plant native to the eastern Mediterranean Basin, is important for its pharmaceutical and seasoning applications. The seed of this plant may be prescribed as an anti-inflammatory agent and its anti-carcinogenic property has recently been under investigation; cumin seeds are used for seasoning purposes in the food science sector. It is adapted to warm and dry warm climatic conditions [20]. The total area devoted to *Cuminum cyminum* cultivation in

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Iran was reported to be 50,000 hectares in 2005 [8].

The thin layer drying procedure has been found to be the most appropriate tool for characterizing the drying parameters [1, 2, 3]. Currently, there are three types of thin layer drying models to describe the drying rate of agricultural products, namely, theoretical, semi-theoretical and empirical models [10, 6]. The theoretical approach concerns either the diffusion equation or simultaneous heat and mass transfer equations. The empirical model neglects the fundamentals of drying processes and presents a direct relationship between average moisture and drying time by means of regression analysis [11]. Also, the semi-theoretical model is a trade-off between the theoretical and empirical ones, derived from a widely used simplification of Fick’s second law of diffusion or modification of the simplified model, such as the Lewis model, the Page model, the Modified Page model and the Henderson model (Table 1).

No work has been reported in developing thin-layer drying equations for *Cuminum cyminum* under multiple air flow rates at various air temperatures. However, numerous investigations have been carried out on different grains and agricultural products [22]. The grain most similar to *Cuminum cyminum* may be rough rice. Basunia and Abe (2001) reported the characteristics of thin-layer natural convection solar drying of rough rice. This work was accomplished at Matsuyama Japan on medium-sized rough rice grains [5].

Full-scale experimentation of dehydration processes for different products is not economically feasible; hence, employing the simulation model for drying rate prediction may be an easy and valuable tool [17].

This study was mainly devoted to the mathematical modeling of forced convection thin layer solar drying for *Cuminum cyminum* seeds.

### MATERIALS AND METHODS

#### Experimental Set up

The experimental set up shown in Figure 1 consisted mainly of a cabinet solar dryer with a sample holding mesh tray (0.22 × 0.33 m, mesh no= 30), a single glazed flat plate air solar collector (0.60 × 1.5 m) and an electrical centrifugal fan (Parma, 1,400 rpm, 50 Hz, Italy). This solar dryer was designed and fabricated in the Agricultural Engineering Department at Shiraz University. Air was drawn through the air collector by the fan for...

### Table 1. Thin layer drying mathematical models.

<table>
<thead>
<tr>
<th>Model no</th>
<th>Model name</th>
<th>Model equation</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Newton</td>
<td>( MR = \exp(-kt) )</td>
<td>Westerman, et al., 1973</td>
</tr>
<tr>
<td>2</td>
<td>Page</td>
<td>( MR = \exp(-kt^*) )</td>
<td>Guarte, 1996</td>
</tr>
<tr>
<td>3</td>
<td>Modified Page</td>
<td>( MR = \exp(-kt)^n )</td>
<td>Yaldiz and Ertkin, 2001</td>
</tr>
<tr>
<td>4</td>
<td>Henderson and Pabis</td>
<td>( MR = a \exp(-kt) )</td>
<td>Yagcioglu et al., 1999</td>
</tr>
<tr>
<td>5</td>
<td>Logarithmic</td>
<td>( MR = a \exp(-kt) + c )</td>
<td>Akpinar et al., 2003</td>
</tr>
<tr>
<td>6</td>
<td>Two term</td>
<td>( MR = a \exp(-k_1t) + b \exp(-k_2t) )</td>
<td>Rahman et al., 1998</td>
</tr>
<tr>
<td>7</td>
<td>Exponential two term</td>
<td>( MR = a \exp(-kt) + (1 - a) \exp(-kat) )</td>
<td>Yaldiz et al., 2001</td>
</tr>
<tr>
<td>8</td>
<td>Wang and Sing</td>
<td>( t = a \ln(MR) + b[\ln(MR)]^2 )</td>
<td>Ozdemir et al., 1999</td>
</tr>
<tr>
<td>9</td>
<td>Thompson</td>
<td>( MR = a \exp(-kt) + (1 - a) \exp(-kbt) )</td>
<td>Yaldiz and Ertekin, 2001</td>
</tr>
<tr>
<td>10</td>
<td>Approximation of diffusion</td>
<td>( MR = a \exp(-kt^*) + bt )</td>
<td>Akpinar et al., 2003</td>
</tr>
<tr>
<td>11</td>
<td>Midilli et al.</td>
<td></td>
<td>Sacilik et al., 2006</td>
</tr>
</tbody>
</table>

*Cuminum cyminum* may be rough rice. Basunia and Abe (2001) reported the characteristics of thin-layer natural convection solar drying of rough rice. This work was accomplished at Matsuyama Japan on medium-sized rough rice grains [5].

Full-scale experimentation of dehydration processes for different products is not economically feasible; hence, employing the simulation model for drying rate prediction may be an easy and valuable tool [17].

This study was mainly devoted to the mathematical modeling of forced convection thin layer solar drying for *Cuminum cyminum* seeds.
Figure 1. The solar dryer system used in this research.

introduction into the drying cabinet. Airflow rates could be altered using a circular damper inserted into the fan discharge. In order to convert the drying system from mixed mode to indirect mode, a thick cover was spread over the glass front wall of the cabinet dryer (Figure 1).

In this research, *Cuminum cyminum* seeds were collected from a farm located in Ferdows city (eastern Iran). The experimental work was carried out on grains of *Cuminum cyminum* in May 2007. The grains were cleaned by hand and stored at a temperature of 4°C in a dry location. A sample of 100 g of stabilized seeds was spread evenly (as a 10 mm thick thin layer) on the mesh tray and placed into the cabinet solar drier. The initial moisture content of the product was 43% (d.b).

**Procedure**

The average initial moisture content of the seeds for all tests was about 43% d.b. and the drying was performed continuously, in each test, for a uniform 90 minutes time period in a solar cabinet dryer to an average moisture content of 8% d.b. Three airflow rates (0.084, 0.127 and 0.155 m³ s⁻¹) were adopted and the experiments were run each sunny day from 11:30 to 13:00. During the course of the experiments, the average solar intensity was 750 W m⁻² (±50 W m⁻²), the ambient air temperature was 27°C (±1°C) and the relative humidity was 30% (±1%). Due to the short drying time period (90 minutes), the inlet air conditions did not experience any appreciable fluctuations. Therefore, the air temperature admitted to the drying chamber was solely dependent upon the rate of airflow through the solar air heater. The lower the airflow rate, the higher was the drying air temperature. Depending upon the airflow rate passing through the solar air heater, the air temperature increases admitted to the dryer ranged from 10 to 23 degrees Celsius.

During the drying experiments, air temperatures were recorded at different locations within the drying system using seven SMT 160 (± 0.5°C) sensors (one at the solar air heater inlet, three in the drying cabinet and three at the air exit of the cabinet dryer) at regular 5 minutes intervals of via a data acquisition system. The solar radiation intensity was measured and recorded at the same time interval using a Casella Pyranometer (0-2,000 W m⁻², 1mv= 1 W m⁻²) placed beside the plane of the collector. The moisture contents of *Cuminum cyminum* samples were measured over different
intervals of time (10, 15, 20, 20 and 25 minutes) using an electrical oven [4]. The airflow rate was measured via a hot wire anemometer (Lutron, Taiwan) located far enough from the fan outlet in a very smooth PVC pipe connected to the dryer air exit.

**Data Analysis**

The moisture ratio (MR) was defined as

\[ MR = \frac{(M - M_e)}{(M_0 - M_e)} \]  

[1]. The values of \( M_e \) are relatively small compared to \( M \) or \( M_0 \) for the drying time, thus the MR can be simplified to \( MR = M/M_0 \) [18]. For investigating the drying characteristics of *Cuminum cyminum*, it is important to model the drying behavior effectively. In this study, the experimental thin layer drying data for *Cuminum cyminum* at different drying air temperatures and flow rates were fitted into 11 commonly used drying models, as shown in Table 1.

To validate the goodness of the fit, three statistical criteria, namely root of mean square error (RMSE), reduced Chi-square (\( \chi^2 \)) and coefficient of determination (R\(^2\)) were calculated using Excel and MSTATC software. The higher the R\(^2\) value and the lower the \( \chi^2 \) and RMSE values, the better is the goodness of fit [22, 23]. Any of these parameters can be calculated as follows [9]:

\[
RMSE = \left[ \frac{1}{N} \sum_{i=1}^{N} (MR_{\text{exp},i} - MR_{\text{pre},i})^2 \right]^{1/2}
\]

(1)

\[
\chi^2 = \sum_{i=1}^{N} \frac{(MR_{\text{exp},i} - MR_{\text{pre},i})^2}{N - p}
\]

(2)

\[
R^2 = \frac{\left[ \sum_{i=1}^{N} (MR_{\text{exp},i} - \overline{MR_{\text{exp}}})(MR_{\text{pre},i} - \overline{MR_{\text{pre}}}) \right]^2}{\sum_{i=1}^{N} (MR_{\text{exp},i} - \overline{MR_{\text{exp}}})^2 \sum_{i=1}^{N} (MR_{\text{pre},i} - \overline{MR_{\text{pre}}})^2}
\]

(3)

**RESULTS AND DISCUSSION**

Thin layers of *Cuminum cyminum* grains were dried in a cabinet solar dryer. Three levels of airflow rates in the dryer cabinet [flow 1= 0.084 (1.15), flow 2= 0.127 (1.75) and flow3= 0.155 (2.05) m\(^3\) s\(^{-1}\) (m s\(^{-1}\))] and two methods of drying (mixed mode type and indirect type) were adapted for the experiments. The initial moisture content of the grains was 43% (d.b) and drying processes were continued to final average moisture contents of 5%-8% (d.b). Drying time durations were kept constant for all drying experiments. Therefore the final moisture content of grains was dependent upon only airflow rates and drying methods.

The moisture content values versus drying time at three airflow rates and for two drying systems are illustrated in Figures 2 and 3.

Refferring to Figures 2 and 3, it can be concluded that the drying processes were carried out in a falling rate period for both
drying methods. This means that the moisture content of the seeds decreases during the drying process, but with a diminishing rate for different airflow rates. These results are in good agreement with the results of other researchers who had some extensive researches on solar drying of different products such as rough rice, pistachio, chili, grapes etc. [5, 22, 23]. For rough rice mixed mode solar drying, drying data were best fitted to the Page model. In this research, the average standard error of estimate for moisture content was reported to be 0.00387 d.b. The two parameters in the Page model, n and k, were linear functions of temperature (T) and drying air relative humidity (RH). The multiple linear regression results for n and k were [5]:

\[ n = 0.68293 + 0.01094T + 0.17036RH, \quad R^2 = 0.80. \]

\[ k = 0.00561 - 0.000073T - 0.0469RH, \quad R^2 = 0.86. \]

The steepest slope in the drying curves occurred with the minimum air flow rate of 0.084 m\(^3\) s\(^{-1}\) because at this low airflow rate, the drying air temperature admitted to the dryer cabinet showed the greatest temperature increase. The effect of the high drying air temperature can be further revealed because the moisture content of the grains was measured to be 5% d.b. at the end of a mixed mode type drying process (90 minutes). In a mixed mode type process, the grain sample was dried not only by the hot air from the solar collector but also due to direct solar radiation received through the transparent cover of the drying cabinet.

The MR values were then fitted against the drying time for the two drying methods by applying the non-linear regression analysis technique. Approximation of The diffusion model demonstrated the best curve fitting results (highest \(R^2\), lowest \(\chi^2\) and RMSE), shown in Table 2 for the mixed mode and in Table 3 for the Midilli model for the indirect type. The approximation of diffusion model was therefore selected to represent the thin layer solar drying characteristics of the mixed mode type, and the Midilli model can be used to predict the characteristics of thin layer solar drying of an indirect type at different drying airflow rates and temperatures for *Cuminum cyminum* grains. There were three coefficients in the approximation of diffusion model and four coefficients in the Midilli model related to the effects of drying air parameters and the material. In order to formulate the relationship between the coefficients and drying air temperatures and airflow rates (corresponding air velocities) for *Cuminum cyminum* grains, a multiple regression analysis method was employed for all data in each solar drying type [15].

In the approximation of diffusion model (mixed mode) the correlation for constants taking into account the drying parameters, drying air temperatures and velocities, were:

<table>
<thead>
<tr>
<th>Model name</th>
<th>Model coefficients</th>
<th>(R^2)</th>
<th>RMSE</th>
<th>(\chi^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newton</td>
<td>(k = 0.054023)</td>
<td>0.952</td>
<td>0.0978</td>
<td>0.0344</td>
</tr>
<tr>
<td>Page</td>
<td>(k = 0.28496, n = 0.455996)</td>
<td>0.988</td>
<td>0.0310</td>
<td>0.0043</td>
</tr>
<tr>
<td>Modified Page</td>
<td>(k = 0.0681, n = 0.5203)</td>
<td>0.990</td>
<td>0.0393</td>
<td>0.0069</td>
</tr>
<tr>
<td>Henderson and Pabis</td>
<td>(a = 0.941654, k = 0.049)</td>
<td>0.937</td>
<td>0.0950</td>
<td>0.0406</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>(a = 0.839717, k = 0.095284, c = 0.157325)</td>
<td>0.994</td>
<td>0.0234</td>
<td>0.0032</td>
</tr>
<tr>
<td>Two term</td>
<td>(a = 0.049, b = 0.470827, k = 0.049, k_u = 0.470827)</td>
<td>0.937</td>
<td>0.0950</td>
<td>0.0813</td>
</tr>
<tr>
<td>Exponential two term</td>
<td>(a = 0.259568, k = 0.149879)</td>
<td>0.960</td>
<td>0.0792</td>
<td>0.0282</td>
</tr>
<tr>
<td>Wang and Sing</td>
<td>(a = 0.03146, b = 0.000253)</td>
<td>0.866</td>
<td>0.1329</td>
<td>0.1060</td>
</tr>
<tr>
<td>Thompson</td>
<td>(a = 0.003778, b = 0.392219)</td>
<td>0.990</td>
<td>0.0324</td>
<td>0.0047</td>
</tr>
<tr>
<td>Approximation of diffusion</td>
<td>(a = 0.766651, k = 0.11179, b = 0.053014)</td>
<td>0.995</td>
<td>0.0199</td>
<td>0.0023</td>
</tr>
<tr>
<td>Midilli et al.</td>
<td>(a = 1.000377, k = 0.174944, n = 0.657723, b = 0.00125)</td>
<td>0.956</td>
<td>0.0213</td>
<td>0.0041</td>
</tr>
</tbody>
</table>

These results are in good agreement with the results of other researchers who had some extensive researches on solar drying of different products such as rough rice, pistachio, chili, grapes etc. [5, 22, 23]. For rough rice mixed mode solar drying, drying data were best fitted to the Page model. In this research, the average standard error of estimate for moisture content was reported to be 0.00387 d.b. The two parameters in the Page model, n and k, were linear functions of temperature (T) and drying air relative humidity (RH). The multiple linear regression results for n and k were [5]:

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The steepest slope in the drying curves occurred with the minimum air flow rate of 0.084 m\(^3\) s\(^{-1}\) because at this low airflow rate, the drying air temperature admitted to the dryer cabinet showed the greatest temperature increase. The effect of the high drying air temperature can be further revealed because the moisture content of the grains was measured to be 5% d.b. at the end of a mixed mode type drying process (90 minutes). In a mixed mode type process, the grain sample was dried not only by the hot air from the solar collector but also due to direct solar radiation received through the transparent cover of the drying cabinet.

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In the approximation of diffusion model (mixed mode) the correlation for constants taking into account the drying parameters, drying air temperatures and velocities, were:
Table 3. Statistical results obtained from various thin layer drying models for indirect mode.

<table>
<thead>
<tr>
<th>Model name</th>
<th>Model coefficients</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newton</td>
<td>$k = 0.035602$</td>
<td>0.915</td>
<td>0.1111</td>
<td>0.0444</td>
</tr>
<tr>
<td>Page</td>
<td>$k = 0.234183$, $n = 0.450329$</td>
<td>0.990</td>
<td>0.0276</td>
<td>0.0034</td>
</tr>
<tr>
<td>Modified Page</td>
<td>$k = 0.0398$, $n = 0.4503$</td>
<td>0.990</td>
<td>0.0276</td>
<td>0.0034</td>
</tr>
<tr>
<td>Henderson and Pabis</td>
<td>$a = 0.894384$, $k = 0.029743$</td>
<td>0.887</td>
<td>0.1016</td>
<td>0.0464</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>$a = 0.775738$, $k = 0.081182$, $c = 0.216791$</td>
<td>0.988</td>
<td>0.0307</td>
<td>0.0056</td>
</tr>
<tr>
<td>Two term</td>
<td>$k_0 = 0.02974$, $a = 0.44719$, $k_1 = 0.02974$, $b = 0.447192$</td>
<td>0.887</td>
<td>0.1016</td>
<td>0.0929</td>
</tr>
<tr>
<td>Exponential two term</td>
<td>$a = 0.222129$, $k = 0.121909$</td>
<td>0.940</td>
<td>0.0862</td>
<td>0.0334</td>
</tr>
<tr>
<td>Wang and Sing</td>
<td>$a = -0.02789$, $b = 0.000218$</td>
<td>0.889</td>
<td>0.1116</td>
<td>0.0747</td>
</tr>
<tr>
<td>Thompson</td>
<td>$a = 0.008813$, $b = 0.337745$</td>
<td>0.988</td>
<td>0.0299</td>
<td>0.0040</td>
</tr>
<tr>
<td>Approximation of diffusion</td>
<td>$a = 0.766617$, $k = 0.11179$, $b = 0.053014$</td>
<td>0.988</td>
<td>0.0199</td>
<td>0.0287</td>
</tr>
<tr>
<td>Midilli et al.</td>
<td>$a = 1.000105$, $k = 0.163617$, $n = 0.600566$, $b = 0.001252$</td>
<td>0.994</td>
<td>0.0225</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

a = -2.088lnv-0.084lnT+1.261v, $R^2 = 0.999$

k = 0.71lnv-0.118lnT-0.415v, $R^2 = 0.986$

b = 0.635lnv+0.109lnT-0.412v, $R^2 = 0.991$

where $T$ is drying air temperature (°C) and $v$ (m s$^{-1}$) is the drying air velocity.

In the Midilli model (indirect type) the correlations for constants were:

$a = 0.024lnv+0.029lnT+0.87$, $R^2 = 0.954$

$b = -0.007lnv-0.011lnT+0.005$, $R^2 = 0.998$

$k = 0.674lnv+0.174lnT-0.479v$, $R^2 = 0.995$

$n = 1.532lnv-0.116lnT+1.075v$, $R^2 = 0.999$

CONCLUSIONS

To find the best mathematical model for Cuminum cyminum thin layer solar drying
by indirect and mixed mode type, a cabinet solar dryer was employed. Three airflow rates (0.084, 0.127 and 0.155 m³ s⁻¹) were adopted and the experiments were run each sunny day from 11:30 to 13:00. Among eleven thin layer mathematical models commonly applied, the approximation of diffusion model and Midilli model showed the best curve fitting results for the experimental moisture ratio values for the mixed mode and indirect type, respectively. The effects of the two drying parameters, air velocity and temperature, on coefficients of the selected mathematical models were investigated and their respective correlation equations were established. The selected thin-layer drying models can be further employed in predicting the design parameters in solar drying processes. Since the cumin seeds are commercially dried in thick-layer dryers, the thin-layer models can be an appropriate starting design criterion for evaluating the thick-layer drying systems.

REFERENCES

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Zomorodian and Moradi

معدل ریاضی خشک کردن خورشیدی لاشه تازه با روش چاپی اجباری در زیره سیز

ع. زمربیان و م. مرادی

چکیده

در این تحقیق مناسبی برای محاسبه خشک کردن خورشیدی لاشه سیز در حال عمیق غیر مستقیم و مختلط انتخاب شد. آزمایشات خشک کردن لاشه در مناسبات اندازه گرفته و رطوبت نسبی ها برای هر برابری به 77 درجه سانتی‌گراد (1σ) و 200٪ (1σ) ثبت گردیدند. در مدت 490 ثانیه مدفوع تابی تری خورشید 250 را بر متر مربع (50 ٪) اندازه‌گیری گردید. به منظور پیدا کردن مناسبی برای مدل ریاضی خشک کردن، نتایج حاصل از آزمایش با یک گروه روش گاردنیون غیر خطی با نمودار مدل مختلف، RMSE و R2 تطبیق داده شدند. نتایج ضریب‌های همبستگی در هر مدل دسته آماده Sپراس چارت‌های آماری X2=155380 و 4588 با Midilli نام مدل سخنگویی . سرینج مرکل RMSE= of Diffusion

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