Resource Allocation in SCMA-based System

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Abstract—In this paper, downlink resource allocation for a sparse code multiple access (SCMA) based heterogeneous cellular networks (HCN) is investigated. The main aim of this paper is to obtain joint power allocation and codebook assignment to achieve the least total system transmit power. In this regard, the proposed objective function is formulated as total transmit power subject to users minimum rate requirement, total available power in each base station (BS), and SCMA constraint. To solve the proposed problem an iterative algorithm considering successive convex approximation (SCA) is applied. Also, convergence of the iterative method is investigated. In the simulation results the effectiveness of proposed resource allocation method are shown.

Index Terms— SCMA, resource allocation, energy efficiency, successive convex approximation (SCA).

I. INTRODUCTION

Due to exploding mobile users and various services, there are many challenges such as more energy consumption, demanding higher data rate, and low latency, which are expected to be addressed by the fifth generation of cellular networks (5G) [1], therefore, new approaches such as suitable multiple access technique, new policy of resource allocation, and appropriate system model should be developed in 5G. Sparse code multiple access (SCMA) is a new candidate of multiple access technique in 5G [3], [4]. SCMA is a non-orthogonal codebook-based multiple-access where each subcarrier can be shared among multiple users by using appropriate codebook assignment, and in the receiver side, each user achieves its signal by applying message passing approach (MPA) [5].

In [6], to improve the SE, SCMA as a multiple access technique has been introduced. To design the SCMA codebooks a systematic approach has been proposed based on design principles networks in [7]. A parallel SCMA-based system has been introduced in [8], which This parallel SCMA system consists of two section: 1) interlayer SCMA encoder and decoder 2) outer-layer mapping. Processing of short codewords are done by the inter-layer SCMA encoder and decoder, each inter-layer short codewords are mapped into a SCMA long codeword.

Heterogeneous cellular networks (HetNet) includes various radio access nodes, such as macro base stations (MBS), femto base stations (FBS), and pico base stations (PBS). HetNet can improve spectrum efficiency (SE) and energy efficiency (EE) by increasing the area spectrum reuse and decreasing the distance between users and access nodes, respectively.

In [9]–[11], the authors proposed a traffic-aware transmission approach to improve the EE of the cellular systems, they have considered that each BS can switch to sleep mode or be shut off mode during off-peak time of traffic loads. In order to minimize total system power consumption an iterative algorithm has been proposed in [12]. As low complexity algorithm in communication networks is important, in [13], the authors proposed an algorithm which can minimize the total down-link transmitted power in multiple input multiple output-orthogonal multiple access (MIMO-OFDMA), with low computational complexity.

In [14], the EE has been used as a metric for link adaptive transmission. In order to measure the efficiency of cellular systems, SE per unit area has been proposed in [15]. In [16], the authors proposed a new concept of area EE to obtain the effects of cell size on the performance of EE.

The main contributions of our paper are summarized as follows:

• We investigate about SCMA-based systems and use the SCMA as a possible multiple access technique in HetNet systems which can reduce total transmit power in a down-link scenario.

• We proposed a novel resource allocation problem in a SCMA-based system to minimize total transmit power subject to minimum rate requirement for each user, and total available power in each cell.

• To solve the proposed problem, we use an iterative algorithm which in each iteration power and codebooks are allocated separately. In each iteration power allocation is non-convex problem and codebook assignment is in integer linear programing (ILP) form. To solve power allocation problem dual method considering successive convex approximation (SCA) is applied and to solve codebook
assignment problem available software for ILP (like opti-toolbox) is used.

The remainder of our paper is organized as follows: In Section II, the system model and problem formulation is presented. Followed by Section III, the solution algorithm is developed. Numerical results are presented in Section IV to evaluate the performance of the proposed resource allocation method. Finally, the conclusions are expressed in Section V.

![System model](image1)

**Fig. 1:** System model.

![SCMA-based system](image2)

**Fig. 2:** SCMA-based system. As can be seen, each codebook contains several sub-carriers, and each of them can be considered as a resource unit like a subcarrier in the OFDMA-based system.

### II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a HetNet system which consists of one MBS, several small base stations (SBS), and SBS and MBS users, which a typical illustration of it is shown in Figure 1. The utilized variables in the system model are summarized in Table I.

The transmitter side in a SCMA-based system contains an encoder which is defined a mapping from \(\log(2K)\) bits to a \(N\)-dimensional codebook of size \(K\) [3]. The codewords are sparse vectors with \(N\)-dimensional. The non-zero entries \((Z < N)\) of this vector is equivalent to specific subcarriers. In the SCMA approach, each codebook includes specific subcarriers, therefore, the number of codebooks is \(C(N, Z) = \binom{N}{Z}\). In Figure 2 we can see an illustration of SCMA approach where each codebook contains multiple subcarriers and each of them can be assigned to multiple users as a subcarrier in a orthogonal multiple access (OFDMA).

In the considered system model, \(q_{m,c}^f\) indicates codebook assignment i.e., if the codebook \(c\) allocates to user \(m\) over BS \(f\) we have \(q_{m,c}^f = 1\), otherwise, \(q_{m,c}^f = 0\). \(p_{m,c}^f\) shows the transmit power to user \(m\) from BS \(f\) over codebook \(c\), and \(h_{m,c}^f\) indicates the channel between user \(m\) and BS \(f\) over codebook \(c\). We assume that the mapping between codebooks and subcarriers are fixed, i.e., \(\rho\) is a known parameter. Note that \(p_{m,c}^f\) is assigned to subcarrier \(n\) in codebook \(c\) based on a given proportion \(\eta_{n,c}\) with \(0 \leq \eta_{n,c} \leq 1\) determined based on codebook design and satisfies \(\sum_{n \in C} \eta_{n,c} = 1 \forall C\) [3]. Consequently, the rate that user \(m\) can achieve from codebook \(c\) over BS \(f\) is given by

\[
r_{m,c}^f = \log(1 + \gamma_{m,c}^f),
\]

where \(\gamma_{m,c}^f\) indicates the SINR which is formulated as

\[
\gamma_{m,c}^f = \frac{\sum_{n=1}^{N} \eta_{n,c} p_{m,c}^f |h_{m,n}^f|^2}{I_{m,c}^f + \sigma_{m,c}^f},
\]

where \(I_{m,c}^f\) (which indicates inter-cell interference) is obtained as

\[
I_{m,c}^f = \sum_{f \in \mathcal{F}/\{f\}} \sum_{m' \in \mathcal{M}_f} \sum_{c' \in \mathcal{C}} p_{m',c'} \eta_{n,c'} |h_{m',n'}|^2.
\]

<table>
<thead>
<tr>
<th>variable</th>
<th>(f)</th>
<th>(f = 1)</th>
<th>(\mathcal{F})</th>
<th>(\mathcal{M}_f)</th>
<th>(\mathcal{N})</th>
<th>(\mathcal{C})</th>
<th>(m)</th>
<th>(\eta_{n,c})</th>
<th>(\rho)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\eta_{n,c})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mathcal{F})</td>
<td>Set of MBS and SBSs, (\mathcal{F} = {1, 2, ..., F}).</td>
<td>(f)</td>
<td>Macro base station</td>
<td>(\mathcal{M}_f)</td>
<td>Set of all users in BS (f), (\mathcal{M}_f = {1, ..., \mathcal{M}_f}).</td>
<td>(\mathcal{N})</td>
<td>Set of all subcarriers, (\mathcal{N} = {1, 2, ..., \mathcal{N}}).</td>
<td>(\mathcal{C})</td>
<td>Set of all Codebooks, (\mathcal{C} = {1, 2, ..., \mathcal{C}}).</td>
<td>(m)</td>
<td>User in BS (f).</td>
<td>(\eta_{n,c})</td>
</tr>
</tbody>
</table>
Finally, the resource allocation problem is formulated as follows:

$$\begin{align*}
\min_{Q, P} & \sum_{f \in F} \sum_{m \in M_f} \sum_{c \in C} q_{m,c}^{f} P_{m,c}^{f}, \\
s.t. & \sum_{m \in M_f} \sum_{c \in C} q_{m,c}^{f} P_{m,c}^{f} \leq P_{max}^{f} \quad \forall f \in F, \\
& \sum_{m \in M_f} \sum_{c \in C} q_{m,c}^{f} P_{m,c}^{f} \leq K, \quad \forall n \in N, f \in F, \\
& \sum_{f \in F} \sum_{c \in C} q_{m,c}^{f} r_{m,c}^{f}(P, Q) \geq R_{min}, \quad \forall m \in M_f, \\
& p_{m,c}^{f} \geq 0, \quad \forall m \in M_f, c \in C, f \in F, \\
& q_{m,c}^{f} \in \{0, 1\}, \quad \forall m \in M_f, c \in C, f \in F,
\end{align*}$$

(4a) - (4f)

where (4b) indicates the maximum available power in each BS, (4c) shows that each subcarrier can be reused at most $K$ times, and (4d) demonstrates minimum rate requirement for each user.

Problem (4) is non-convex and contains both integer and continuous variables, therefore, the available resource allocation method to solve convex problem cannot be directly applied.

III. SOLUTION OF THE JOINT POWER AND CODEBOOK ALLOCATION

To solve problem (4), we use an iterative approach which, in each iteration, power and codebook are allocated separately. Based on this iterative approach, power is allocated in each iteration considering fixed codebook (which are assigned based on previous iteration), then, codebook are assigned with fixed power. The algorithm is continued until $|vec(P(s)) - vec(P(s-1))| \leq \Theta$ be satisfied. An overview of the described algorithm is summarize in Algorithm 1.

A. Initialization Method

The initialization method for problem (4) is shown in Algorithm 2 [19], [20].

C. Power Allocation

The power allocation problem in each iteration is

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**Algorithm 1** ITERATIVE RESOURCE ALLOCATION ALGORITHM

1: Initialization: Set $s = 0$ ($s$ shows the iteration number), compute $Q(0)$ and $P(0)$ by Algorithm (2).
2: Set $Q = Q(s)$, apply SCA approach,
3: use $p_{m,c}^{f} = \exp(\tilde{p}_{m,c}^{f})$ to approximate constraint (4e) by a convex function.
4: Solve (6) and assign its solution in to $P(s + 1)$,
5: Solve (5) and assign its solution in to $Q(s + 1)$,
6: If $|vec(P(s)) - vec(P(s - 1))| \leq \Theta$
   stop,
   else
   set $s = s + 1$ and go back to 2

**Algorithm 2** INITIALIZATION METHOD

**COMPUTING P(0):**
Consider SBSs ($f = 2, \ldots, F$) are not transmitting, then for MBS ($f = 1$) users solve the convex problem of finding $P(0)$ (problem (4) without rate requirement constraint and interference).

**FINDING Q(0):**
Codebooks are assigned to MBS user which has the highest SINR.

B. codebook assignment

The codebook assignment problem in each iteration is expressed as

$$\begin{align*}
\min_{Q} & \sum_{f \in F} \sum_{m \in M_f} \sum_{c \in C} q_{m,c}^{f} P_{m,c}^{f}, \\
s.t. & \sum_{m \in M_f} \sum_{c \in C} q_{m,c}^{f} P_{m,c}^{f} \leq P_{max}^{f} \quad \forall f \in F, \\
& \sum_{f \in F} \sum_{c \in C} q_{m,c}^{f} r_{m,c}^{f}(P, Q) \geq R_{min}, \quad \forall m \in M_f, \\
& q_{m,c}^{f} \in \{0, 1\}, \quad \forall m \in M_f, c \in C, f \in F,
\end{align*}$$

(5a) - (5d)

This problem is an integer linear programming (ILP) problem which can be solved by using available software for it. We utilize opti-toolbox [18] to solve this problem.
Algorithm 3 POWER ALLOCATION ALGORITHM
1: Initialization: Set $I_r = 0$, and initialize $P_0(s) = P(s)$, $\alpha = 1$ and $\beta = 0$.
2: Update $P_n(s)$ by solving (6a),
3: Update $\alpha$ and $\beta$,
4: If $I_r = I_r_{\text{max}}$ or convergence stop,
   else
   set $I_r = I_r + 1$ and go back to 2
5: Output: $P(s + 1) = P_{\text{max}}(s)$

formulated as

$$\min_{P} \sum_{f \in F} \sum_{m \in M_f \cap \epsilon \in C} q^f_{m,c} p^f_{m,c}.$$  \hfill (6a)

s.t. $\sum_{m \in M_f \cap \epsilon \in C} q^f_{m,c} p^f_{m,c} \leq p_{\text{max}}$, $\forall f \in F$,

$$\sum_{f \in F} q^f_{m,c} r^h_{m,c}(P,Q) \geq R_{\text{min}}, \forall m \in M_f,$$

$$p^f_{m,c} \geq 0, \forall m \in M_f, c \in C, f \in F,$$

$$q^f_{m,c} \in \{0,1\}, \forall m \in M_{f,c}, c \in C, f \in F.$$  \hfill (6c)–(6e)

To tackle the non-convexity issue of constraint (6c), successive convex approximation with low complexity (SCALE) [17] is applied which approximate (6c) by a convex function. In SCALE approach the following inequality is used

$$\log_2(1 + z) \geq \alpha \log_2 z + \beta,$$  \hfill (7)

where $\alpha$ and $\beta$ are obtain as:

$$\alpha = \frac{z}{1 + z}, \beta = \log_2(1 + z) - \alpha \log_2(z).$$  \hfill (8)

where $z$ and $\bar{z}$ are SINR in $n^{th}$ and $n-1^{th}$ iteration, respectively. By applying (7), (6c) is written by

$$\sum_{f \in F} \sum_{\epsilon \in C} q^f_{m,c} \left[ \alpha^f_{m,c} \log_2(\gamma_{m,c}) + \beta^f_{m,c} \right] \geq R_{\text{min}}.$$  \hfill (9)

Constraint (9) is still non-convex, by transforming $p^f_{m,c} = \exp(\tilde{p}^f_{m,c})$, (9) reformulate by (10) which convexity of each term of (10) is clear. Consequently, the convex optimization problem is reformulate as follows:

$$\min_{\tilde{P}} \sum_{f \in F} \sum_{m \in M_f \cap \epsilon \in C} q^f_{m,c} \exp(\tilde{p}^f_{m,c}).$$  \hfill (11a)

s.t. $\sum_{m \in M_f \cap \epsilon \in C} q^f_{m,c} \exp(\tilde{p}^f_{m,c}) \leq p_{\text{max}}$, $\forall f \in F$,

$$\sum_{f \in F} \sum_{\epsilon \in C} q^f_{m,c} \left[ \alpha^f_{m,c} \log_2(\gamma^f_{m,c}) + \beta^f_{m,c} \right] \geq R_{\text{min}},$$  \hfill (11b)

$$\exp(\tilde{p}^f_{m,c}) \geq 0, \forall m \in M_f, c \in C, f \in F,$$

$$q^f_{m,c} \in \{0,1\}, \forall m \in M_f, c \in C, f \in F.$$  \hfill (11d)

Then, to achieve appropriate approximation Algorithm 3 is applied. The lagrangian function of optimization problem (11), is expressed as

$$L(\tilde{P}, \lambda, \delta) = \sum_{f \in F} \sum_{m \in M_f \cap \epsilon \in C} q^f_{m,c} \exp(\tilde{p}^f_{m,c}) +$$

$$\sum_{i \in M} \lambda_i (p_{\text{max}} - m \in M_f \cap \epsilon \in C) \sum_{i \in M} q^f_{m,c} \exp(\tilde{p}^f_{m,c}) + \sum_{i \in M} \delta_i \left( \sum_{f \in F} \sum_{\epsilon \in C} q^f_{m,c} \left[ \alpha^f_{m,c} \log_2(\gamma^f_{m,c}) + \beta^f_{m,c} \right] - R_{\text{min}} \right).$$

(12)

The dual objective function is given by

$$g(\lambda, \delta) = \max_{\tilde{P}} L(\tilde{P}, \lambda, \delta).$$  \hfill (13)

To solve the dual problem, we should find the stationary point of (12), with respect to $\tilde{p}$, and with $\{\lambda, \delta\}$ fixed. Therefore, we have (14).

Then, after simplifying (14), and applying transformation $p_{m,c} = \exp(\tilde{p}_{m,c})$, $\tilde{p}_{m,c}$, $\forall m, f, c$ is given by (17). The dual variables are updated through sub gradient method by applying the following formula

$$\lambda^{u+1} = [\lambda^u - \varepsilon_1 (p_{\text{max}} - \sum_{m \in M_f \cap \epsilon \in C} q^f_{m,c} \exp(\tilde{p}^f_{m,c}))]^+,$$

$$\delta^{u+1} = [\delta^u - \varepsilon_2 \left( \sum_{f \in F} \sum_{\epsilon \in C} q^f_{m,c} r^f_{m,c} - R_{\text{min}} \right)]^+.$$  \hfill (15)

where $[\Phi]^+ = \max(0, \Phi)$, $u$ indicates the iteration number, and $\varepsilon_i$ shows the small step-size to update the lagrange variables.

**Proposition 1:** The SCALE approximation converges to a locally optimal solution.

**Proof:** After iteration $s$, we have $P = P(s)$, $\alpha = \alpha(s)$ and $\beta = \beta(s)$. Let $G_m(P, \beta, \alpha) = \sum_{f \in F} \sum_{\epsilon \in C} p^f_{m,c} r^f_{m,c}, \forall m$. It can be shown that solution of each iteration is a feasible solution for the next iteration [21] and [22]. For each user, we have $R_{\text{min}}^m =$
\[
\sum_{f \in F} \sum_{c \in C} q_{m,c}^f \left[ \alpha_{m,c}^f (\bar{p}_{m,c}^f + \log(\sum_{n=1}^N \eta_{n,c}^f |h_{m,n}^f|^2)) - \log(\sum_{f \in F} (\sum_{m \in M_f} \sum_{c \in C} I_{m,c}^f + (\sigma_{m,c}^f)^2)) + \beta_{m,c}^f \right].
\]

(10)

\[
d(\hat{\mathbf{P}}, \lambda, \delta) = 0 \Rightarrow q_{\text{max}}^f \exp(\hat{p}_{m,c}^f) - \lambda_f q_{\text{max}}^f \exp(\hat{p}_{m,c}^f) - \eta_{m,c}^f |h_{m,n}^f|^2 \exp(\hat{p}_{m,c}^f) \sum_{i \in M_f} \delta_i \left( \sum_{j \in F_0} \sum_{r \in C} \sum_{n=1}^N \eta_{m,c}^f \eta_{n,c}^r |h_{m,n}^r|^2 \right) + \delta_m q_{m,c}^f \alpha_{m,c}^f = 0.
\]

(14)

\[
p_{m,c}^f = \left[ \frac{\delta_m \alpha_{m,c}^f}{\lambda_f - 1 + \eta_{m,c}^f |h_{m,n}^f|^2 \sum_{i \in M_f} \delta_i \left( \sum_{j \in F_0} \sum_{r \in C} \sum_{n=1}^N \eta_{m,c}^f \eta_{n,c}^r |h_{m,n}^r|^2 \right)} \right]^{+}.
\]

(17)

Algorithm 4 FIND STATIONARY POINT

1: Initialization: Set \( u = 0 \), find \( \lambda_0^f \) and \( \delta_0^f \) by uniform method.
2: Compute \( p \) by using (17),
3: Update \( \lambda \) and \( \delta \) by using (15) and (16) respectively,
4: If \( \| \delta^u - \delta^{u-1} \| \leq \varepsilon \) and \( \| \lambda^u - \lambda^{u-1} \| \leq \varepsilon \)
      stop,
   else
      set \( u = u + 1 \) and go back to 2.

\( G_m(\mathbf{P}(s-1), \beta(s-1), \alpha(s-1)) \) because all users rate constraints are active at the optimal solution of the considered problem (see Lemma 2 in [21]), from (7) we have \( R_{\text{valid}} = G_m(\mathbf{P}(s-1), \beta(s-1), \alpha(s-1)) \leq G_m(\mathbf{P}(s-1)) \), then, based on the update step of \( \alpha \) and \( \beta \) in Algorithm 3 we have \( R_{\text{min}} = G_m(\mathbf{P}(s-1), \beta(s-1), \alpha(s-1)) \leq G_m(\mathbf{P}(s-1)) = G_m(\mathbf{P}(s-1), \beta(s), \alpha(s)) \) (see [22] and Theorem 1 in [21]). Therefore, after each iteration, the value of \( G \) either improves or stays unaltered as the previous iteration value. Consequently, the SCALE approximation converges to the last feasible solution.

IV. SIMULATION RESULTS

The numerical results are presented under various system parameters to evaluate the performance of our resource allocation policy. The diameter of MBS and SBSs are supposed .5 Km and 10 m, respectively, and the other parameters are considered as: \( N = 8 \), \( Z = 2 \), \( F = 4 \), \( P_{\text{max}}^f = 2 \), \( \forall f \in F \), \( P_{\text{max}}^1 = 20 \), \( K = 6 \), \( \eta_{m,c}^f = 1/2 \), and \( h_{m,c}^f = x_{m,c}^f (d_{m,c}^f)^{\xi} \), where \( \xi \) indicates the path loss exponent which is considered \( \xi = -2 \), \( x_{m,c}^f \) representing the Rayleigh fading, and \( d_{m,c}^f \) shows the distance between user \( m \) and BS \( f \). Fig.3 shows total transmit power versus the users minimum rate requirement considering \( M = 8 \) (total number of users), and Fig.4 presents total transmit power versus the number of users.

A. Receiver complexity of SCMA and OFDMA

1) SCMA: The complexity order of MPA method is given by \( O(I_T(\phi^{d}))) \) [23], where \( \phi \) indicates the codebook set size, \( I_T \) denotes the total number of iterations, and \( d \) denotes the non-zero elements in each row of the matrix \( \mathbf{J} \) where \( \mathbf{J} = (\mathbf{J}_1, \ldots, \mathbf{J}_n) \) is the factor graph matrix. In other word, \( d \) is the maximum number of signals superimposed on each subcarrier.

2) OFDMA: In OFDMA-based system by applying MMSE, the estimated signal is given by \( \hat{x} = \mathbf{D} \mathbf{y} \) where \( \mathbf{D} \) is the transformation matrix calculated as \( \mathbf{D} = \min_{\mathbf{D}} \mathbb{E} [||\mathbf{x} - \mathbf{D} \mathbf{y}||^2] \), whose solution is given by \( \mathbf{D} = (\mathbf{H}^H \mathbf{H} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}^H \). Consequently, the complexity order of OFDMA receiver is approximately given by \( O(I_T^2) \).

As the numerical results show by applying SCMA as the multiple access technique, the total transmit power is decreased by approximately \( \%25 \), at the cost of increasing complexity of the receiver and transmitter.

V. CONCLUSION

In this paper, we investigated about power allocation and codebook assignment in a SCMA-based HCN system to achieve the least total system power consumption considering minimum rate requirement for each user and SCMA constraint. To solve the corresponding optimization problem we used an iterative approach which power and codebooks are assigned separately and algorithm
is continued until convergence. In each iteration, we applied SCA approach to solve the power allocation problem and we used opti-toolbox to solve the codebook assignment problem. As the numerical results show, the proposed policy to resource allocation in SCMA-based system outperforms the OFDMA-based system at the cost of increasing complexity.

REFERENCES